Abington Heights School District
Calculus AP BC Curriculum

In Calculus AP BC, students develop their numeracy skills through the following areas of study:

1. Limits and Continuity
2. Differentiation and its Applications
3. Integration and its Applications
4. Parametric Equations, Polar Coordinates, Vector-Valued Functions
5. Infinite Sequences and Series

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Review Date:

AH MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where collaborative problem solving and perseverance lead to excellence

Abington Heights Math Framework

Stakeholders	Actions
Students	★ Engage in mathematical discussions, share their ideas openly, be inquisitive, seek to understand and learn more about mathematical concepts, and try their best daily. ★ Exhibit creativity and curiosity in problem solving individually and collaboratively. ★ Persevere in engaging and challenging daily mathematical practice. ★ Come prepared to learn every day.
Teachers	★ Create a safe and collaborative classroom environment where students feel vested in a shared vision for mathematical excellence. ★ Develop high quality instruction that meets the needs of all learners through differentiation. ★ Use a variety of 21st century methodologies to advance learning. ★ Partner with parents and guardians to support student success. ★ Establish a collaborative community within the building and amongst grade levels to ensure a cohesive level of instruction.
Building Leaders	★ Deeply understand the needs of teachers, students, the instructional materials being used, programs being implemented, and the expectations for state-level assessment scores ○ Knowledgeable about program and grade level standards ○ Ensure consistent and equal access to high-quality instructional materials and resources, building. ★ Be partners with teachers, students and families: ○ Provide guidance and support to the mathematical community. ○ Understand needs of teachers, students and families. ★ Trust the educators to make professional decisions based on program, student, and district needs.
Central Admin	★ Effectively communicate to the school board and community specific areas of need and how to support teachers and building leaders in a quest for mathematical excellence ★ Deeply understand the needs of teachers, students, the instructional materials being used, programs being implemented, and the expectations for state-level assessment scores ○ Have a common metric for mathematical excellence. ○ Ensure consistent and equal access to high-quality instructional materials and resources, district. ○ Re-examine best practices/curriculum routinely (6 years). ★ Support a culture of collaboration between the other stakeholder groups to maintain the standard of excellence of the Abington Heights ★ Trust the educators to make professional decisions based on program, student, and district needs.
Parents/ Community	★ Be a strong support system and contribute by building a positive math community for students. ★ Encourage a positive math mindset. ★ Have conversations with their children about school and ask what they are learning about in school. ★ Be open, receptive to the district's ideas about student learning and reach out to teachers/school to learn more about how they can support. ★ Trust the educators to make professional decisions based on program, student, and district needs.
School Board	★ Provide the fiscal resources to support: ○ Highly qualified professionals for mathematics ○ High-quality instructional materials ○ Effective and efficient math interventions for remediation ○ Professional development for math content and instructional practices ★ Trust the educators to make professional decisions based on program, student, and district needs.

Calculus AP BC Scope and Sequence

Month	Unit	Estimated Number of Weeks
September	Limits and Continuity	2
	Differentiation: Definition and Fundamental Properties	2
October	Differentiation: Composite, Implicit, and Inverse Functions	3
	Contextual Applications of Differentiation	1
November	Contextual Applications of Differentiation	2
	Analytical Applications of Differentiation	2
December	Analytical Applications of Differentiation	2
	Integration and Accumulation of Change	2
January	Integration and Accumulation of Change	3
	Differential Equations	2
February	Applications of Integration	3
	Parametric Equations, Polar Coordinates, and Vector-Valued Functions	1
March	Parametric Equations, Polar Coordinates, and Vector-Valued Functions	3
	Infinite Sequences and Series	1
April	Infinite Sequences and Series	3
May	AP Exam and Final Exam Preparation	4
June	AP Exam and Final Exam Preparation	1

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 1: Limits and Continuity	What is a limit? What do we use limits for? How is the limit of a function different from its value? How can limits be found algebraically, graphically, and using tables? What are left and right-handed limits? What are limits at infinity and how are they related to horizontal asymptotes? What is the value of an end behavior model for understanding the behavior of a function? What are the different types of function discontinuity?	1.1 Introducing Calculus: Can Change Occur at an Instant? 1.2 Defining Limits and Using Limit Notation 1.3 Estimating Limit Values from Graphs 1.4 Estimating Limit Values from Tables 1.5 Determining Limits Using Algebraic Properties of Limits 1.6 Determining Limits Using Algebraic Manipulation 1.7 Selecting Procedures for Determining Limits 1.8 Determining Limits Using the Squeeze Theorem 1.9 Connecting Multiple Representations of Limits 1.10 Exploring Types of Discontinuities	Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant Represent limits analytically using correct notation Estimate limits of functions Determine the limits of functions using limit theorems Determine the limits of functions using equivalent expressions for the function or the squeeze theorem Justify conclusions about continuity at a point using the definition Determine intervals over which a function is continuous	Google slides Unit 1 Notes Packet Unit 1 Homework Packet Construct open top box with largest volume using piece of cardstock (by trial and error as introduction to course) Desmos	Delta Math Quiz 1-1 Quiz 1-2 Unit 1 Test AP Classroom Unit 1 Free Response Questions (FRQs) [handout/online] AP Classroom Unit 1 Multiple Choice Questions (MCQs) [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 1: Limits and Continuity (continued)	What are the implications of the Intermediate Value Theorem?	1.11 Defining Continuity at a Point 1.12 Confirming Continuity over an Interval 1.13 Removing Discontinuities 1.14 Connecting Infinite Limits and Vertical Asymptotes 1.15 Connecting Limits at Infinity and Horizontal Asymptotes 1.16 Working with the Intermediate Value Theorem (IVT)	Determine values of x or solve for parameters that make discontinuous functions continuous, if possible Interpret the behavior of functions using limits involving infinity Explain the behavior of a function on an interval using the Intermediate Value Theorem		
Unit 2: Differentiation: Definition and Fundamental Properties	What is a derivative? How is the average rate of change of a function related to the slope? How can limits be used to find the instantaneous rate of change of a function at a point? How can derivatives be calculated?	2.1 Defining Average and Instantaneous Rates of Change at a Point 2.2 Defining the Derivative of a Function and Using Derivative Notation 2.3 Estimating Derivatives of a Function at a Point	Determine average rates of change using difference quotients Represent the derivative of a function as the limit of a difference quotient Determine the equation of a line tangent to a curve at a given point Estimate derivatives	Google slides Unit 2 Notes Packet Unit 2 Homework Packet	Delta Math Quiz 2 Unit 2 Test AP Classroom Unit 2 FRQs [handout/online] AP Classroom Unit 2 MCQs [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 2: Differentiation : Definition and Fundamental Properties (continued)	What is the formal definition of a derivative? How can derivatives of polynomial functions be calculated using the formal definition? What can you learn about the shape of a graph by finding its derivative at a point? When is a function differentiable? What are the power, product, and quotient rules, and how can they be applied to find derivatives? How is the derivative related to velocity and acceleration? What are the derivatives of the trigonometric and logarithmic functions?	2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist 2.5 Applying the Power Rule 2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple 2.7 Derivatives of cos x,sin x, e^x, and In x 2.8 The Product Rule 2.9 The Quotient Rule 2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions	Explain the relationship between differentiability and continuity Calculate derivatives of polynomial functions using the power rule Calculate derivatives of transcendental functions (trigonometric exponential, and logarithmic) Calculate derivatives of products and quotients of differentiable functions		

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 3: Differentiation : Composite, Implicit, and Inverse Functions	What is the chain rule, and how can it be applied to find derivatives? What is implicit differentiation? How is the derivative of an inverse function related to the derivative of the original function?	3.1 The Chain Rule 3.2 Implicit Differentiation 3.3 Differentiating Inverse Functions 3.4 Differentiating Inverse Trigonometric Functions 3.5 Selecting Procedures for Calculating Derivatives 3.6 Calculating Higher Order Derivatives	Calculate derivatives of compositions of differentiable functions Calculate derivatives of implicitly defined functions Calculate derivatives of inverse and inverse trigonometric functions Determine higher order derivatives of a function	Google slides Unit 3 Notes Packet Unit 3 Homework Packet Desmos	Delta Math Quiz 3 Unit 3 Test AP Classroom Unit 3 FRQs [handout/online] AP Classroom Unit 3 MCQs [online]
Unit 4: Contextual Applications of Differentiation	What are derivatives used for? How is the derivative related to velocity and acceleration? How can the tangent line be used to approximate the function value? What are related rates problems?	4.1 Interpreting the Meaning of the Derivative in Context 4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration 4.3 Rates of Change in Applied Contexts Other Than Motion 4.4 Introduction to Related Rates 4.5 Solving Related Rates Problems	Interpret the meaning of a derivative in context Calculate rates of change in applied contexts Interpret rates of change in applied contexts Calculate related rates in applied contexts Interpret related rates in applied contexts	Google slides Unit 4 Notes Packet Unit 4 Homework Packet	Delta Math Quiz 4 Unit 4 Test AP Classroom Unit 4 FRQs [handout/online] AP Classroom Unit 4 MCQs [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 4: Contextual Applications of Differentiation (continued)	How can derivatives be used to solve multi-variable problems where several variables change with respect to time?	4.6 Approximating Values of a Function Using Local Linearity and Linearization 4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms	Approximate a value on a curve using the equation of a tangent line Determine limits of functions that result in indeterminate forms		
Unit 5: Analytical Applications of Differentiation	What are derivatives used for? What are the implications of the Mean Value Theorem? What are extrema and how can the derivative be used to find them? What is concavity and how is it related to the second derivative? How can the first and second derivatives be used as aides in graphing functions? How can the derivative be applied to optimization problems?	5.1 Using the Mean Value Theorem 5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points 5.3 Determining Intervals on Which a Function Is Increasing or Decreasing 5.4 Using the First Derivative Test to Determine Relative (Local) Extrema 5.5 Using the Candidates Test to Determine Absolute (Global) Extrema 5.6 Determining Concavity of Functions over Their Domains	Justify conclusions about functions by applying the Mean Value Theorem over an interval Justify conclusions about functions by applying the Extreme Value Theorem Justify conclusions about the behavior of a function based on the behavior of its derivatives Calculate minimum and maximum values in applied contexts or analysis of functions Interpret minimum and maximum values values calculated in applied contexts	Google slides Unit 5 Notes Packet Unit 5 Homework Packet Construct closed top box with maximum volume using given piece of cardstock (using optimization) - as follow up to Unit 1 activity Desmos	Delta Math Quiz 5 Unit 5 Test AP Classroom Unit 5 FRQs [handout/online] AP Classroom Unit 5 MCQs [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 5: Analytical Applications of Differentiation (continued)		5.7 Using the Second Derivative Test to 3 Determine Extrema 5.8 Sketching Graphs of Functions and Their Derivatives 5.9 Connecting a Function, Its First Derivative, and Its Second Derivative 5.10 Introduction to Optimization Problems 5.11 Solving Optimization Problems 5.12 Exploring Behaviors of Implicit Relations	Determine critical points of implicit relations Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives		
Unit 6: Integration and Accumulation of Change	What is integration? How can Rectangular Approximation methods be used to estimate areas? What is a definite integral?	6.1 Exploring Accumulations of Change 6.2 Approximating Areas with Riemann Sums 6.3 Riemann Sums, Summation Notation, and Definite Integral Notation	Interpret the meaning with the graph of a rate of change in context Approximate a definite integral using geometry and numerical methods Interpret the limiting case of the Riemann sum as definite integral	Google slides Unit 6 Notes Packet Unit 6 Homework Packet	Delta Math Quiz 6 Unit 6 Test AP Classroom Unit 6 FRQs [handout/online] AP Classroom Unit 6 MCQs [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 6: Integration and Accumulation of Change (continued)	What geometric methods can be used to calculate integrals? How can the trapezoid rule be used to estimate areas? How is the antiderivative related to the definite and indefinite integral?	6.4 The Fundamental Theorem of Calculus and Accumulation Functions 6.5 Interpreting the Behavior of Accumulation Functions Involving Area 6.6 Applying Properties of Definite Integrals 6.7 The Fundamental Theorem of Calculus and Definite Integrals 6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation 6.9 Integrating Using Substitution 6.10 Integrating Functions Using Long Division and Completing the Square 6.11 Integrating Using Integration by Parts 6.12 Using Linear Partial Fractions	Represent the limiting case of the Riemann sum as a definite integral Represent accumulation functions using definite integrals Calculate a definite integral using areas and properties of definite integrals Evaluate definite integrals analytically using the Fundamental Theorem of Calculus Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives For integrands requiring substitution or rearrangements into equivalent forms: (a) determine indefinite integrals and (b) evaluate definite integrals		

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 6: Integration and Accumulation of Change (continued)		6.13 Evaluating Improper Integrals 6.14 Selecting Techniques for Antidifferentiation	For integrals requiring integration by parts: (a) determine indefinite integrals and (b) evaluate definite For integrands requiring integration by linear partial fractions: (a) determine indefinite integrals and (b) evaluate definite Evaluate an improper integral or determine that an improper integral diverges		
Unit 7: Differential Equations	In what ways are differential equations related to real world problems? How do slope fields relate to differential equations?	7.1 Modeling Situations with Differential Equations 7.2 Verifying Solutions for Differential Equations 7.3 Sketching Slope Fields 7.4 Reasoning Using Slope Fields 7.5 Approximating Solutions Using Euler's Method	Interpret verbal statements of problems as differential equations involving a derivative expression Verify solutions to differential equations Estimate solutions to differential equations Determine particular solutions to differential equations Determine general solutions to differential equations	Google slides Unit 7 Notes Packet Unit 7 Homework Packet Desmos	Delta Math Quiz 7 Unit 7 Quiz AP Classroom Unit 7 FRQs [handout/online] AP Classroom Unit 7 MCQs [online] WebWork - BC Midterm Review Midterm Exam

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 7: Differential Equations (continued)		7.6 Finding General Solutions Using Separation of Variables 7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables 7.8 Exponential Models with Differential Equations 7.9 Logistic Models with Differential Equations	Determine general and particular solutions for problems involving differential equations in context Interpret the meaning of a differential equations and its variables in context Interpret the meaning of the logistical growth model in context		
Unit 8: Applications of Integration	Why do we learn to integrate? How is the area beneath a curve related to the displacement of an object in motion? How can integrals be used to find the area between two curves? How can the disk and shell methods be used to find the volumes and densities of solids?	8.1 Finding the Average Value of a Function on an Interval 8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals 8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts 8.4 Finding the Area Between Curves Expressed as Functions of x	Determine the average value of a function using definite integrals Determine values for positions and rates of change using definite integrals in problems involving motion Interpret the meaning of a definite integral in accumulation problems Determine net change using definite integrals in applied contexts	Google slides Unit 8 Notes Packet Unit 8 Homework Packet Desmos	Delta Math Quiz 8 Unit 8 Test AP Classroom Unit 8 FRQs [handout/online] AP Classroom Unit 8 MCQs [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 8: Applications of Integration (continued)	What is the relationship between the definite integral and the accumulation of a quantity? What is the formula for the length of a curve?	8.5 Finding the Area Between Curves Expressed as Functions of y 8.6 Finding the Area Between Curves That Intersect at More Than Two Points 8.7 Volumes with Cross Sections: Squares and Rectangles 8.8 Volumes with Cross Sections: Triangles and Semicircles 8.9 Volume with Disc Method: Revolving Around the x- or y-Axis 8.10 Volume with Disc Method: Revolving Around Other Axes 8.11 Volume with Washer Method: Revolving Around the x- or y-Axis 8.12 Volume with Washer Method: Revolving Around Other Axes	Calculate areas in the plane using the definite integral Calculate volumes of solids with known cross sections using definite integrals Calculate volumes of solids of revolution using definite integrals Determine the length of a curve in the plane defined by a function, using a definite integral		

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 8: Applications of Integration (continued)		8.13 The Arc Length of a Smooth, Planar Curve and Distance Traveled			
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions	What are parametrically defined functions? What are the advantages to parametric and polar representation over Cartesian representation? How are parametrically defined functions differentiated and integrated? What are vector-valued functions and how are they differentiated and integrated? How are functions expressed in a polar coordinate system and how are they differentiated and integrated?	9.1 Defining and Differentiating Parametric Equations 9.2 Second Derivatives of Parametric Equations 9.3 Finding Arc Lengths of Curves Given by Parametric Equations 9.4 Defining and Differentiating Vector-Valued Functions 9.5 Integrating Vector-Valued Functions 9.6 Solving Motion Problems Using Parametric and Vector-Valued Functions 9.7 Defining Polar Coordinates and Differentiating in Polar Form	Calculate derivatives of parametric equations Determine the length of a curve in the plane defined by parametric functions, using a definite integral Calculate derivatives of vector-valued functions Determine a particular solution given a rate vector and initial conditions Determine values for positions and rates of change in problems involving planar motion Calculate derivatives of functions written in polar coordinates Calculate areas of regions defined by polar curves by using definite integrals	Google slides Unit 9 Notes Packet Unit 9 Homework Packet Desmos	Delta Math Quiz 9 Unit 9 Test AP Classroom Unit 9 FRQs [handout/online] AP Classroom Unit 9 MCQs [online]

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions (continued)		9.8 Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve 9.9 Finding the Area of the Region Bounded by Two Polar Curves			
Unit 10: Infinite Sequences and Series	What is an infinite series and when does it converge or diverge? What is a power series and how is its radius and interval of convergence calculated? What is an Error Bound and how is it calculated? What is a Taylor series and how can a Taylor polynomial be used to approximate any function?	10.1 Defining Convergent and Divergent Infinite Series 10.2 Working with Geometric Series 10.3 The nth Term Test for Divergence 10.4 Integral Test for Convergence 10.5 Harmonic Series and p-Series 10.6 Comparison Tests for Convergence 10.7 Alternating Series Test for Convergence 10.8 Ratio Test for Convergence	Determine whether a series converges or diverges Approximate the sum of a series Represent a function at a point as a Taylor polynomial Approximate function values using a Taylor polynomial Determine the error bound associated with a Taylor polynomial approximation Determine the radius of convergence and interval of convergence for a power series Represent a function as a Taylor series or a Maclaurin series	Google slides Unit 10 Notes Packet Unit 10 Homework Packet Desmos Vase Project - model side of vase with functions and use calculus to determine volume and surface area (check volume with water) [or other project after AP exam]	Delta Math Quiz 10 Unit 10 Test AP Classroom Unit 10 FRQs [handout/online] AP Classroom Unit 10 MCQs [online] WebWork: The Big BC Review Part 1 WebWork: The Big BC Review Part 2 Final Exam Part 1 - No Calculator Final Exam Part 2 - Calculator Required

Unit	Essential Questions	Content	Skills	Activities	Assessment/Evidence of Learning
Unit 10: Infinite Sequences and Series (continued)		10.9 Determining Absolute or Conditional Convergence 10.10 Alternating Series Error Bound 10.11 Finding Taylor Polynomial Approximations of Functions 10.12 Lagrange Error Bound 10.13 Radius and Interval of Convergence of Power Series 10.14 Finding Taylor or Maclaurin Series for a Function 10.15 Representing Functions as Power Series	Interpret Taylor series and Maclaurin series Represent a given function as a power series		

Portrait of an Abington Heights Mathematician

By the end of Calculus, students will:

Limits and Continuity	Derivatives	Applications of Derivatives	Analyzing Functions	Integrals
☐ Estimate limits from graphs and tables ☐ Evaluate limits by algebraic manipulation ☐ Analyze graphs to determine one-sided limits	☐ Determine the derivative of a function using the limit of the difference quotient ☐ Find derivatives of functions using the product, quotient, power, and/or chain rules ☐ Evaluate the derivatives of trigonometric, exponential, and logarithmic functions ☐ Determine a higher order derivative for a given function ☐ Apply the process of implicit differentiation	☐ Use derivatives to solve related rates problems ☐ Use calculus-methods to determine optimal values ☐ Solve real-life optimization problems	☐ Use derivatives to sketch a curve by obtaining critical values of a function, classifying as relative or absolute minima/maxima, identifying inflection points, and analyzing function to determine increasing and decreasing intervals	☐ Integrate polynomials, trigonometric, exponential, and logarithmic functions ☐ Investigate properties of indefinite and definite integration ☐ Integrate with U-substitution